

## Basics of Engineering Economy

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# Basics of Engineering Economy 

Second Edition

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## BASICS OF ENGINEERING ECONOMY, SECOND EDITION

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## Dedication

To our grandchildren Abigail, Benjamin, Grace, Taryn, and Tyler

May they lead us in successfully meeting the challenges of tomorrow

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## Preface

A11 of the basic principles, techniques, and tools of undergraduate engineering economics are covered in this second edition. The textual material, examples, and problems are designed to meet the needs of a two- or three-semester/ quarter credit hour service course for all disciplines of engineering, engineering technology, and engineering management. The printed and electronic versions are suitable for different course formats. Especially helpful are the website-based podcasts, which incorporate voice-over animated and annotated PPT slides. These podcasts serve as supplemental and support materials for students in any course formatresident, online, or distance education.

## FEATURES — NEW AND OLD

Notable enhancements for this edition are in the areas of new or upgraded topics, teaching and learning aids, and website materials.

## New Topics

- Ethics and its important connection to economic decisions are introduced in Chapter 1 and discussed further in Chapter 7 on public-sector projects.
- Risk analysis is expanded with its own new Chapter 14, which also introduces simulation with random sampling utilizing simple spreadsheet functions.
- External rate of return material is expanded with the modified ROR method (MIRR) and return on invested capital (ROIC) methods covered, along with hand solution and spreadsheet illustrations.


## New Teaching and Learning Aids

- The final answer to every third end-of-chapter problem is presented in a new Appendix C. Full solutions are available on the website.
- Financial calculator usage is included throughout the formative chapters, with an introduction to calculator-based solutions presented in Chapter 2 in parallel with spreadsheet functions.
- Tax tables for corporations and individuals utilize the same format as IRS tables.
- Spreadsheet screen shots are streamlined and more colorful for clearer understanding of content and the approach to problem solution.
- Using Spreadsheets (Appendix A) is updated to Excel ${ }^{\oplus} 2010$.
- End-of-chapter problems number in excess of 870 with approximately $2 / 3$ of them new or rewritten for this edition.
- Solutions using factors, calculator, and spreadsheet functions are presented for selected examples and end-of-chapter problems throughout the formative chapters.


## New Website Materials

- Podcasts, incorporating voice-over animated and annotated PPT slides that summarize the essential topics are available on the website (www.mhhe.com/ blank). (An icon in the margin of the text identifies material included in these podcasts.)
- A detailed solution is available in open access form for every end-of-chapter problem that has its final answer in Appendix C.
- Live spreadsheets are available for all examples, plus an image library of all tables and figures in the text.

The familiar features that make this an easy-to-use and quick-to-learn-from text continue to be included.

- Purpose statement and learning outcomes at the beginning of each chapter with outcomes tied to individual sections.
- Examples in each section, taken from different engineering disciplines, are maintained from the first edition, plus several new examples to better illustrate current topics and solution approaches.
- Spreadsheet and calculator applications are primarily concentrated in a final section of each chapter, allowing incorporation of electronic solutions or omission of this technology, at the discretion of the professor.
- Large number of end-of-chapter problems cover all aspects of the text's material in each section.
- Multiple-choice questions for each chapter are useful in a review for the FE Exam. Alternatively, these problems can be used as additional problems or for review prior to a course exam. These can be easily incorporated into auto-grade systems for online and distance-learning course structures through course management systems such as Blackboard.
- Solutions manual, lecture slides, and image library of figures are available online for each chapter with password protection for adopters.


## USES OF TEXT

The writing style emphasizes brief, crisp coverage of principles, techniques, and alternative selection guidelines based on time-value-of-money computations. This book is developed in order to reduce the time necessary to present, grasp, and apply the essentials of engineering economic analysis. Most chapters that cover the fundamentals of the subject include hand, calculator, and spreadsheet solutions. More complex solutions that utilize a spreadsheet are separately shown in a final section of each chapter.

Students should have attained a sophomore or higher level standing to thoroughly understand the engineering context of the techniques and problems addressed. A background in calculus is not necessary; however, a basic familiarity
with engineering terminology in a student's own engineering discipline makes the material more meaningful and, therefore, easier to learn and apply.

The text may be used in a wide variety of ways in an undergraduate coursefrom a few weeks that introduce the basics of engineering economics, to a full twoor three-semester/quarter credit hour course. For senior students who have little or no background in engineering economic analysis in earlier courses, this text provides an excellent senior-level introduction as the senior project is designed and developed.

Engineering economy is one of the few engineering topics that is equally applicable to both individuals and corporate and government employees. It can analyze personal finances and investments in a fashion similar to corporate project finances. Students will find that this text serves well as a reference throughout their courses and senior design projects, and especially after graduation as a reference source in engineering project work.

Because various engineering curricula concentrate on different aspects of engineering economics, sections and chapters can be covered or skipped to tailor the text's usage in print or electronic forms. For example, cost estimation that is often of more importance to chemical engineering is concentrated in a special chapter. Public sector economics for civil engineering is discussed separately. After-tax analysis, cost of capital, and decision-making under risk are introduced for industrial and systems engineering and engineering management curricula that include a shortened course in engineering economy. Examples treat areas for electrical, petroleum, mechanical, and other engineering disciplines.

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We welcome comments and corrections that will improve this text or its online learning materials. Our e-mail addresses are lelandblank@yahoo.com and atarquin@utep.edu.

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## Foundations of Engineering Economy



Mike Kemp/Rubberball/Getty Images

The need for engineering economy is primarily motivated by the work that engineers do in performing analysis, synthesizing, and coming to a conclusion as they work on projects of all sizes. In other words, engineering economy is at the heart of making decisions. These decisions involve the fundamental elements of cash flows of money, time, and interest rates. This chapter introduces the basic concepts and terminology necessary for an engineer to combine these three essential elements in organized, mathematically correct ways to solve problems that will lead to better decisions.

## Purpose: Understand and apply the fundamental concepts and terminology of engineering economy.

## LEARNING OUTCOMES

1. Determine the role of engineering economy in the decisionmaking process.
2. Identify what is needed to successfully perform an engineering economy study.
3. Perform calculations about interest rate and rate of return.
4. Understand what equivalence means in economic terms.
5. Calculate simple interest and compound interest for one or more interest periods.
6. Identify and use engineering economy terminology and symbols.
7. Understand cash flows, their estimation, and how to graphically represent them.
8. Formulate spreadsheet and calculator functions used in engineering economy.
9. Describe the terms universal and personal morals, and professional ethics; understand the Code of Ethics for Engineers.

### 1.1 WHAT IS ENGINEERING ECONOMY?

Before we begin to develop the fundamental concepts upon which engineering economy is based, it is appropriate to define the term engineering economy. In the simplest of terms, engineering economy is a collection of techniques that simplify comparisons of alternatives on an economic basis. In defining what engineering economy is, it might also be helpful to define what it is not. Engineering economy is not a method or process for determining what the alternatives are. On the contrary, engineering economy begins only after the alternatives have been identified. If the best alternative is actually one that the engineer has not even recognized as an alternative, then all of the engineering economic analysis tools in this book will not result in its selection.

Engineering economic analysis is able to answer professional and personal financial questions. If you wish to evaluate the economics of purchasing a new home or leasing versus buying a new automobile for yourself, the techniques of engineering economy covered in this text are just as applicable as they are for determining if a replacement piece of equipment should be purchased by your employer.

While economics will be the sole criterion for selecting the best alternatives in this book, real-world decisions usually include many other factors in the decisionmaking process. For example, in determining whether to build a nuclear-powered, gas-fired, or coal-fired power plant, factors such as safety, air pollution, public acceptance, water demand, waste disposal, global warming, and many others would be considered in identifying the best alternative. The inclusion of other factors (besides economics) in the decision-making process is called multiple attribute analysis. This topic is introduced in Chapter 14.

### 1.2 PERFORMIING AN ENGINEERING ECONOMY STUDY

In order to apply economic analysis techniques, it is necessary to understand the basic terminology and fundamental concepts that form the foundation for engineeringeconomy studies. Some of these terms and concepts are described below.

### 1.2.1 Alternatives

An alternative is a stand-alone solution for a given situation. We are faced with alternatives in virtually everything we do, from selecting the method of transportation we use to get to work every day to deciding between buying a house or renting one. Similarly, in engineering practice, there are always several ways of accomplishing a given task, and it is necessary to be able to compare them in a rational manner so that the most economical alternative can be selected. The alternatives in engineering considerations usually involve such items as purchase cost (first cost), anticipated useful life, yearly costs of maintaining assets (annual maintenance and operating costs), anticipated resale value (salvage value), and the interest rate. After the facts and all the relevant estimates have been collected, an engineering economy analysis can be conducted to determine which is best from an economic point of view.

### 1.2.2 Cash Flows

The estimated inflows (revenues and savings) and outflows (costs) of money are called cash flows. These estimates are truly the heart of an engineering economic analysis. They also represent the weakest part of the analysis, because most of the numbers are judgments about what is going to happen in the future. After all, who can accurately predict the price of oil next week, much less next month, next year, or next decade? Thus, no matter how sophisticated the analysis technique, the end result is only as reliable as the accuracy of the data that it is based on. This means that economic decisions about proposed alternatives are made under risk, that is, without certainty. Techniques that utilize sensitivity analysis, risk analysis, and multiple attribute analysis assist in understanding the consequences of variation in cash flow estimates.

### 1.2.3 Alternative Selection

Every situation has at least two alternatives. In addition to the one or more formulated alternatives, there is always the alternative of inaction, called the do-nothing $(D N)$ alternative. This is the as-is or status quo condition. In any situation, when one consciously or subconsciously does not take any action, he or she is actually selecting the DN alternative. Of course, if the status quo alternative is selected, the decision-making process should indicate that doing nothing is the most favorable economic outcome at the time the evaluation is made. The procedures developed in this book will enable you to consciously identify the alternative(s) that is (are) economically the best.

### 1.2.4 Evaluation Criteria

Whether we are aware of it or not, we use criteria every day to choose between alternatives. For example, when you drive to campus, you decide to take the "best" route. But how did you define best? Was the best route the safest, shortest, fastest, cheapest, most scenic, or what? Obviously, depending upon which criterion or combination of criteria is used to identify the best, a different route might be selected each time. In economic analysis, financial units (dollars or other currency) are generally used as the tangible basis for evaluation. Thus, when there are several ways of accomplishing a stated objective, the alternative with the lowest overall cost or highest overall net income is selected.

### 1.2.5 Intangible Factors

In many cases, alternatives have noneconomic or intangible factors that are difficult to quantify. When the alternatives under consideration are hard to distinguish economically, intangible factors may tilt the decision in the direction of one of the alternatives. A few examples of noneconomic factors are safety, customer acceptance, reliability, convenience, and goodwill.

### 1.2.6 Time Value of Money

It is often said that money makes money. The statement is indeed true, for if we elect to invest money today, we inherently expect to have more money in the future.

If a person or company borrows money today, by tomorrow more than the original loan principal will be owed. This fact is also explained by the time value of money.

## The change in the amount of money over a given time period is called the time value of money; it is the most important concept in engineering economy.

The time value of money can be taken into account by several methods in an economy study, as we will learn. The method's final output is a measure of worth, for example, rate of return. This measure is used to accept or reject an alternative.

### 1.3 INTEREST RATE, RATE OF RETURN, AND MARR

Interest is the manifestation of the time value of money, and it essentially represents "rent" paid for use of the money. Computationally, interest is the difference between an ending amount of money and the beginning amount. If the difference is zero or negative, there is no interest. There are always two perspectives to an
 amount of interest-interest paid and interest earned. Interest is paid when a person or organization borrows money (obtains a loan) and repays a larger amount. Interest is earned when a person or organization saves, invests, or lends money and obtains a return of a larger amount. The computations and numerical values are essentially the same for both perspectives, but they are interpreted differently.

Interest paid or earned is determined by using the relation

$$
\begin{equation*}
\text { Interest }=\text { end amount }- \text { original amount } \tag{1.1}
\end{equation*}
$$

When interest over a specific time unit is expressed as a percentage of the original amount (principal), the result is called the interest rate or rate of return (ROR).

$$
\text { Interest rate or rate of return }=\frac{\text { interest accrued per time unit }}{\text { original amount }} \times 100 \% ~[1.2]
$$

The time unit of the interest rate is called the interest period. By far the most common interest period used to state an interest rate is 1 year. Shorter time periods can be used, such as, $1 \%$ per month. Thus, the interest period of the interest rate should always be included. If only the rate is stated, for example, $8.5 \%$, a 1 -year interest period is assumed.

The term return on investment (ROI) is used equivalently with ROR in different industries and settings, especially where large capital funds are committed to engineering-oriented programs. The term interest rate paid is more appropriate from the borrower's perspective, while rate of return earned is better from the investor's perspective.

An employee at LaserKinetics.com borrows \$10,000 on May 1 and must repay a total of $\$ 10,700$ exactly 1 year later. Determine the interest amount and the interest rate paid.

## Solution

The perspective here is that of the borrower since $\$ 10,700$ repays a loan. Apply Equation [1.1] to determine the interest paid.

$$
\text { Interest paid }=\$ 10,700-10,000=\$ 700
$$

Equation [1.2] determines the interest rate paid for 1 year.

$$
\text { Percent interest rate }=\frac{\$ 700}{\$ 10,000} \times 100 \%=7 \% \text { per year }
$$

a. Calculate the amount deposited 1 year ago to have $\$ 1000$ now at an interest rate of 5\% per year.
b. Calculate the amount of interest earned during this time period.

## Solution

a. The total amount accrued (\$1000) is the sum of the original deposit and the earned interest. If $X$ is the original deposit,

$$
\begin{aligned}
\text { Total accrued } & =\text { original amount }+ \text { original amount (interest rate) } \\
\$ 1000 & =X+X(0.05)=X(1+0.05)=1.05 X
\end{aligned}
$$

The original deposit is

$$
X=\frac{1000}{1.05}=\$ 952.38
$$

b. Apply Equation [1.1] to determine interest earned.

$$
\text { Interest }=\$ 1000-952.38=\$ 47.62
$$

In Examples 1.1 and 1.2 the interest period was 1 year, and the interest amount was calculated at the end of one period. When more than one interest period is involved (e.g., if we wanted the amount of interest earned after 3 years in Example 1.2), it is necessary to state whether the interest is accrued on a simple or compound basis from one period to the next. Simple and compound interest will be discussed in Section 1.5.

Engineering alternatives are evaluated upon the prognosis that a reasonable rate of return (ROR) can be realized. A reasonable rate must be established so that the accept/reject decision can be made. This reasonable rate, called the minimum attractive rate of return (MARR), is the lowest interest rate that will induce companies or individuals to invest their money. The MARR must be higher than the cost of money used to finance the alternative, as well as higher than the rate that would be expected from a bank or safe (minimal risk) investment. Figure 1.1 indicates the relations between different rates of return. In the United States, the current U.S. Treasury bill rate of return is sometimes used as the benchmark safe rate.

For a corporation, the MARR is always set above its cost of capital, which is the interest rate a company must pay for capital funds needed to finance projects.


For example, if a corporation can borrow capital funds at an average of 5\% per year and expects to clear at least $6 \%$ per year on a project, the MARR will be at least $11 \%$ per year.

The MARR is also referred to as the hurdle rate; that is, a financially viable project's expected ROR must meet or exceed the hurdle rate. Note that the MARR is not a rate calculated like the ROR; MARR is established by financial managers and is used as a criterion for accept/reject decisions. The following inequality must be correct for any accepted project.

$$
\text { ROR } \geq \text { MARR }>\text { cost of capital }
$$

Descriptions and problems in the following chapters use stated MARR values with the assumption that they are set correctly relative to the cost of capital and the expected rate of return. If more understanding of capital funds and the establishment of the MARR is required, refer to Section 13.5 for further detail.

An additional economic consideration for any engineering economy study is inflation. In simple terms, bank interest rates reflect two things: a so-called real rate of return plus the expected inflation rate. The safest investments (such as government bonds) typically have a $3 \%$ to $4 \%$ real rate of return built into their overall interest rates. Thus, an interest rate of, say, $9 \%$ per year on a government bond means that investors expect the inflation rate to be in the range of $5 \%$ to $6 \%$ per year. Clearly, then, inflation causes interest rates to rise. Inflation is discussed in detail in Chapter 10.

### 1.4 EQUIVALENCE



Equivalent terms are used often in the transfer between scales and units. For example, 1000 meters is equal to (or equivalent to) 1 kilometer, 12 inches equals 1 foot, and 1 quart equals 2 pints or 0.946 liter.

In engineering economy, when considered together, the time value of money and the interest rate help develop the concept of economic equivalence, which means that different sums of money at different times would be equal in economic value. For example, if the interest rate is $6 \%$ per year, $\$ 100$ today (present time) is equivalent to $\$ 106$ one year from today.

$$
\text { Amount in one year }=100+100(0.06)=100(1+0.06)=\$ 106
$$

So, if someone offered you a gift of \$100 today or \$106 one year from today, it would make no difference which offer you accepted from an economic perspective. In either case you have $\$ 106$ one year from today. However, the two sums of money are equivalent to each other only when the interest rate is $6 \%$ per year. At a higher or lower interest rate, $\$ 100$ today is not equivalent to $\$ 106$ one year from today.

In addition to future equivalence, we can apply the same logic to determine equivalence for previous years. A total of $\$ 100$ now is equivalent to $\$ 100 / 1.06=$ $\$ 94.34$ one year ago at an interest rate of $6 \%$ per year. From these illustrations, we can state the following: $\$ 94.34$ last year, $\$ 100$ now, and $\$ 106$ one year from now are equivalent at an interest rate of $6 \%$ per year. The fact that these sums are equivalent can be verified by computing the two interest rates for 1 -year interest periods.

$$
\frac{\$ 6}{\$ 100} \times 100 \%=6 \% \text { per year }
$$

and

$$
\frac{\$ 5.66}{\$ 94.34} \times 100 \%=6 \% \text { per year }
$$

Figure 1.2 indicates the amount of interest each year necessary to make these three different amounts equivalent at $6 \%$ per year.

FIGURE 1.2
Equivalence of three amounts at a 6\% per year interest rate.


AC-Delco makes auto batteries available to General Motors dealers through privately owned distributorships. In general, batteries are stored throughout the year, and a 5\% cost increase is added each year to cover the inventory carrying charge for the distributorship owner. Assume you own the City Center Delco facility. Make the calculations necessary to show which of the following statements are true and which are false about battery costs.
a. The amount of $\$ 98$ now is equivalent to a cost of $\$ 105.60$ one year from now.
b. A truck battery cost of $\$ 200$ one year ago is equivalent to $\$ 205$ now.
c. A $\$ 38$ cost now is equivalent to $\$ 39.90$ one year from now.
d. A $\$ 3000$ cost now is equivalent to $\$ 2887.14$ one year ago.
e. The carrying charge accumulated in 1 year on an investment of $\$ 2000$ worth of batteries is $\$ 100$.

## Solution

a. Total amount accrued $=98(1.05)=\$ 102.90 \neq \$ 105.60$; therefore, it is false. Another way to solve this is as follows: Required original cost is $105.60 / 1.05=\$ 100.57 \neq \$ 98$.
b. Required old cost is $205.00 / 1.05=\$ 195.24 \neq \$ 200$; therefore, it is false.
c. The cost 1 year from now is $\$ 38(1.05)=\$ 39.90$; true.
d. Cost one year ago is $3000 / 1.05=\$ 2857.14 \neq 2887.14$; false.
e. The charge is $5 \%$ per year interest, or $\$ 2000(0.05)=\$ 100$; true.

### 1.5 SIMPLE AND COMPOUND INTEREST

The terms interest, interest period, and interest rate were introduced in Section 1.3 for calculating equivalent sums of money for one interest period in the past and one period in the future. However, for more than one interest period, the terms simple interest and compound interest become important.

Simple interest is calculated using the principal only, ignoring any interest accrued in preceding interest periods. The total simple interest over several periods is computed as

$$
\begin{equation*}
\text { Interest }=(\text { principal })(\text { number of periods)(interest rate }) \tag{1.3}
\end{equation*}
$$

where the interest rate is expressed in decimal form. Therefore, the total (future) amount accumulated after several periods is the principal plus interest over all $n$ periods.

HP borrowed money to do rapid prototyping for a new ruggedized computer that targets desert oilfield conditions. The loan is $\$ 1$ million for 3 years at $5 \%$ per year simple interest. How much money will HP repay at the end of 3 years? Tabulate the results in $\$ 1000$ units.

## EXAMPLE 1.3

